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Dependence of wave front refraction on pupil size due to the presence of higher order aberrations

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PURPOSE. *Propagation of light through the optical pathway within the eye can lead to a deformation of the wave front that might affect objective but also subjective refraction depending on pupil size. The aim of this study was to investigate the change in wave front refraction that is calculated on the basis of second order Zernike polynomials when varying the pupil size from 6 mm to 3 mm. The change was correlated with the amount of fourth and sixth order spherical aberration and fourth and sixth order astigmatism.*
METHODS. *Wave front aberrations were measured in 130 eyes by means of a Tscherning wave front sensor at a pupil size of 6 mm. Wave front aberrations in terms of Zernike coefficients up to sixth order were approximated for 6 mm and 3 mm pupil size. The wave front refraction was calculated based on the second order Zernike coefficients for both pupil diameters. Resulting differences in wave front refraction (sphere or cylinder) due to the change in pupil size were correlated with the initial higher order aberrations determined for the 6.0 mm pupil by means of a linear regression (Spearman rank correlation coefficient).*
RESULTS. *The correlation between the change in sphere and cylinder on one hand and the spherical aberration and higher order astigmatism on the other hand was found to be highly significant ($p<0.001$), with a correlation coefficient of $R = 0.96$ for sphere and $R = 0.85$ for cylinder.*
CONCLUSIONS. *Calculating the wave front refraction on the basis of second order Zernike polynomials is plagued with the influence of the higher order aberration preexisting in the individual eye. This is one reason why this method does not represent precisely enough subjective refraction. Other methods that calculate the refraction based on wave front measurement independent from the pupil size should be established in the ophthalmic community. (Eur J Ophthalmol 2005; 15: 680-7)*

KEY WORDS. *Pupil, Pupil size, Aberration, Wave front aberration, Zernike*

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INTRODUCTION

There are different ways of calculating the best corrective lens for the refraction of an individual eye from objective wave front aberrations (1-3). One way that has been frequently used is to calculate the lens that minimizes the wave front variance (1). This is achieved by minimizing the second order Zernike coefficients (4-12) with sphero-cylindrical corrections.

The amount of refraction will change with the pupil size and the amount of change will be associated with the amount of higher order aberrations, particularly fourth order spherical aberration (3). This aberration can lead to an improvement of image quality (differ-

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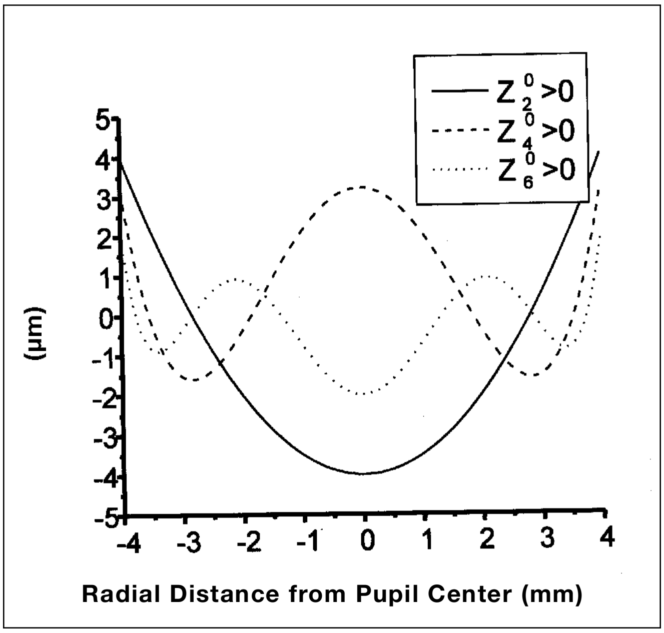


Fig. 1 - *Schematic drawing that represents the compensation mechanism of higher order spherical aberrations. The difference of fourth and sixth order spherical aberration* ($Z_0^4 - Z_0^6$) *is represented by the solid line. The differences of these two polynomials entail the largest and also relevant proportion of the refraction modification with different pupil diameters. In this case these polynomials work with unequal signs strengthening.*

ent higher order aberrations are compensating each other) or might decrease image quality (aggravating each other) (3).

The change in wave front refraction (second Zernike order) is a mathematical fact. In brief, Zernike polynomials two radial order up have a component with the same radial order (4-12). Therefore, one might expect a transformation from higher to lower Zernike polynomials when using a small pupil. For example, fourth order spherical aberration (r^4–radial order) contains in the second order a defocus component (r^2–radial order) with reversed sign. Thus, the central part of a fourth order spherical aberration results in defocus when the pupil is belittled.

The Zernike coefficient change with pupil size and, therefore, the wave front refraction calculated only on the second order Zernike coefficient will change in dependence to the amount of higher order aberrations preexistent in the particular eye.

Among others, we have realized that wave front refraction calculated only on second Zernike order polynomials are confusing for most ophthalmologists, especially when eyes with large higher order aberrations are investigated.

So far, the amount of change due to the pupil size variation is unknown for normal subjects and will depend on the amount of higher order aberration of the particular eye. This is of clinical relevance as the wave refraction calculated on the basis of second order Zernike coefficients has become a common technique for the determination of objective refraction in a series of commercial wave front devices (Industry answers Questions: 5th International Congress on Wave front Sensing and Optimized Refractive Surgery; Whistler, Canada; February 21–23, 2004).

Hence, the knowledge of how this objective refraction can be affected by the higher order aberrations with respect to the actual pupil size should be recognized by the users of wave front devices, in particular when customized treatments are planned.

The aim of our study was to investigate change in the amount of wave front refraction caused by higher order aberrations when varying the pupil size from

TABLE I - DEMOGRAPHIC AND REFRACTION DATA FROM INCLUDED PATIENTS

		Means ± SD	**Range**
Age		32.9±11.3 yer	18 to 63 yars
Sex	Female	82 (63.1%)	
	Male	48 (36.9%)	
Eyes	Right	64 (49.2%)	
	Left	66 (50.8%)	
Refraction	Sphere	-1.7±2.16 D	- 7.50 to + 2.25
	Cylinder	- 0.44±0.49 D	0 to - 2.75

6 mm amount of refraction change that might be observed in clinical routine. Here, the wave front refraction is calculated on the basis of second order Zernike coefficients as it is done by most of the commercially available wave front sensors. The calculated changes in wave front sphere and cylinder were correlated with fourth and sixth order spherical aberration and fourth and sixth order astigmatism, respectively.

METHODS

A total of 130 eyes of 90 individuals were enrolled in this study; bilateral measurements were available in 40 persons (13). Subjects (eyes) were eligible to be included in the study if 1) they were at least 18 years of age, 2) they were free of ocular diseases, 3) they had a best-corrected visual acuity of 20/20 or better, 4) the spherical equivalent of the refraction was between – 8.0 D and + 2.0 D, (5) the manifest refractive cylinder was less than 3.0 D (Tab. I).

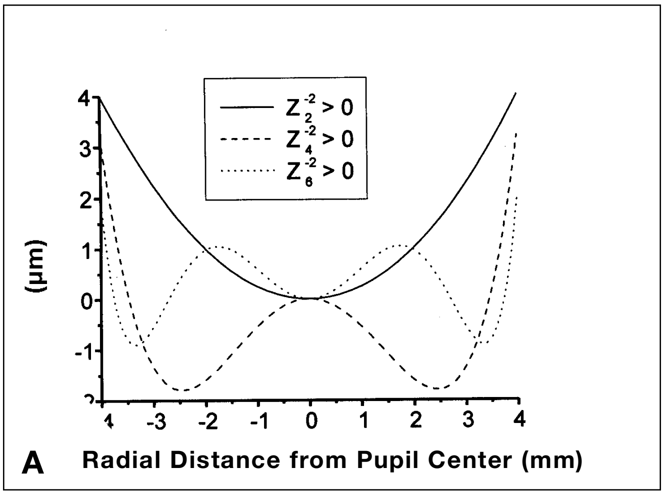
The subjective refraction was performed at a pupil size of 3 mm. Wave front aberrations were measured by means of a Tscherning wave front sensor known as Dresden Wave front Analyzer (14-16). All measurements were performed after the pupils were dilated by one drop of tropicamide 1%. Data were accepted when the pupil size was 6.0 mm or larger during the measurement and all the spots on the retina were detected. The wavelength used for wave front sensing was 532 nm.

Wave front aberrations in terms of Zernike coefficients up to sixth order were approximated for 6 mm and 3 mm diameter of the exit pupil, respectively.

The Zernike coefficients presented in our study must be divided by the appropriate normalization factor F_n and multiplied by the pupil radius to convert them into the Zernike representations proposed by the VSIA taskforce (17). Here the normalization factors are determined by:

$$F_{n,m} = \begin{cases} \sqrt{2(n+1)} & \text{if } n-2m \neq 0 \\ \sqrt{(n+1)} & \text{if } n-2m = 0 \end{cases}$$

where n is the order of the Zernike monomial and m is the azimuthal frequency of the term.



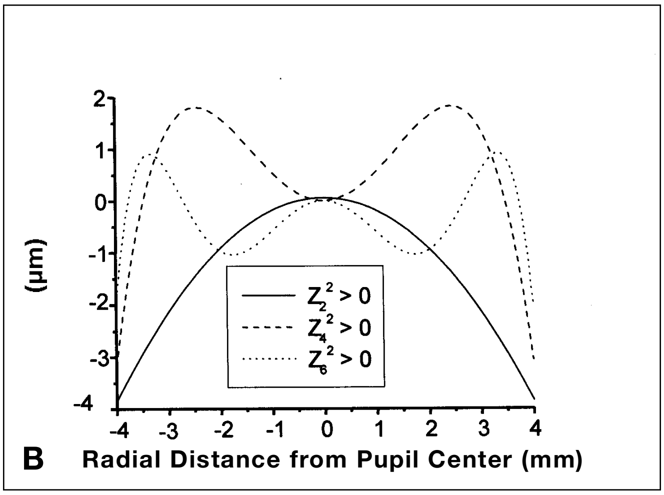


Fig. 2 - *Schematic drawing that represents the compensation mechanism of higher order astigmatism at 0° and 90°.* **(A)** *Influence of fourth and sixth order cylindrical aberration with different radial distance from pupil center (value 0°).* **(B)** *Influence of fourth and sixth order cylindrical aberration with different radial distance from pupil center (value 90°).*

The Taylor defocus and astigmatism terms are often converted into conventional sphere, cylinder, and a cylinder axis as it can be corrected by ophthalmic lenses (1).

$$Cyl = 2\sqrt{(W_3 - W_5)^2 + W_4^2} \qquad [1]$$

$$Sph = W_3 + W_5 - \frac{C}{2} \qquad [2]$$

with

$$W_4 = \frac{\left(2Z_2^0 + Z_2^2 - 6Z_4^0 - 3Z_4^2 + 12Z_6^0 + ...\right)}{R} \qquad [3]$$

$$W_5 = \frac{\left(2Z_2^{-2} + 6Z_4^{-2} + 12Z_6^{-2} + ...\right)}{R} \qquad [4]$$

$$W_6 = \frac{\left(2Z_2^0 - Z_2^2 - 6Z_4^0 + 3Z_4^2 + 12Z_6^0 + ...\right)}{R} \qquad [5]$$

The Taylor defocus represents the location of the paraxial image plane (paraxial defocus).

The Zernike terms Z_2^{-2} , Z_2^0, and Z_2^2 are left over in equations 3–5 after conventional defocus is used in the balancing of higher orders. As mentioned above the wave front refraction can be calculated based on the second order Zernike coefficients for both pupil diameters as follows:

$$Cyl = \frac{4\sqrt{Z_2^{-2} + Z_2^2}}{R} \qquad Sph = \frac{4Z_2^0}{R} - \frac{1}{2}Cyl \qquad \alpha = \frac{1}{2}\arctan\left(\frac{Z_2^{-2}}{Z_2^2}\right)$$

Here, R stands for the radius of the exit pupil of the eye. If $Z_2^2 \geq 0$, then α (degrees) must be changed according to α = 90 - α to obtain axis notation as used to report refraction data in ophthalmology. The signs of sphere (Sph) and cylinder (Cyl) are reversed to obtain ophthalmic correction. The wave front refraction calculated only on the second order Zernike modes represents the defocus and cylinder after the smallest wave front variance is achieved by some spherocylindrical correction.

Resulting differences in wave front refraction (sphere or cylinder) due to the change in pupil size were correlated with the initial higher order aberrations determined for the 6.0 mm pupil by means of a linear regression (Spearman rank correlation coefficient). A commercially available software package (Origin 6.0, Mircocal Inc.) was used for data analysis.

Sphere was correlated with the difference of fourth and sixth order spherical aberration ($Z_4^0 - Z_6^0$) This difference of the two polynomials entails the largest and, therefore, relevant proportion of the refraction modification with different pupil diameters. In this case these polynomials work strengthening with unequal signs (Fig. 1). These higher orders were chosen because they also contain second order astigmatism in their polynomials as can also be seen from equations 3 to 5. Other higher aberrations above sixth Zernike order might also affect the second order astigmatism when the pupil is delimited. However, the amounts of such aberrations are very small in normal eyes and, therefore, neglected in our investigations.

Cylinder was correlated with a value obtained from the fourth order polynomials. Theoretically the polynomials Z_4^{-2}, Z_4^{2} , Z_6^{-2}, and Z_6^{2} have the potential to influence the wave front cylinder as can be seen from equations 3 to 5. However, for further calculations only the aberrations of fourth order were applied, because of the small influence of the sixth order aberrations. These correlations were assumed to be significant because of the rotational symmetry of the Zernike polynomials for defocus, fourth and sixth order spherical aberration, and the characteristics of second- and fourth-order astigmatism (Fig. 2, a and b).

The outcome of this analysis is a collection of numbers, Zernike coefficients that represent the magnitude of different types of aberrations in the eye. Three of these aberration coefficients are familiar to clinicians, since they describe the sphere, cylinder, and axis of the cylinder. Other coefficients indicate the amount of so-called higher order aberrations, including coma and spherical aberration.

RESULTS

The correlation between the change in wave front sphere and the spherical aberration ($Z_4^0 - Z_6^0$) in Figure 3 was found to by highly significant ($p<0.001$) with a correlation coefficient of R = 0.96. The normal distribution of the sphere difference between 6 mm and 3 mm pupil has a standard deviation of ±0.27 D and a range from -0.5 D to +0.9 D (Fig. 4). The mean difference in sphere was determined to be 0.16 D.

The correlation between the change in cylinder and the higher order astigmatism ($Z_4^2 - Z^{-2}_6$) in Figure 5 was found to by highly significant ($p<0.001$) with a correlation coefficient of R = 0.85. The normal distribution of the cylinder difference between 6 mm and 3 mm pupil has a standard deviation of ±0.155 D and a range from -0.4 to 0.45 D (Fig. 6). The mean difference in cylinder was determined to be 0.043 D.

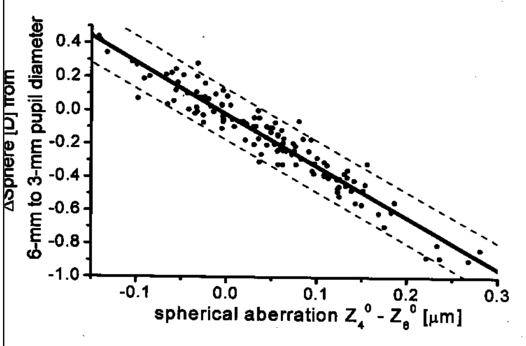


Fig. 3 - *Correlation between change in sphere and higher order spherical aberration. The correlation coefficient was R = 0.96 with a significance level of p<0.001.*

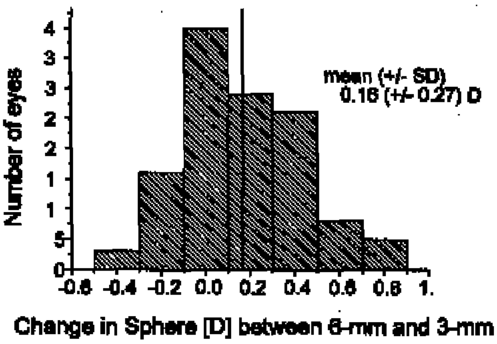


Fig. 4 - *Histogram of the differences in sphere all measured eyes when changing the pupil diameter from 3 to 6 mm. Approximately 95% of the investigated eyes have a change in sphere up to 1.4 D when reducing the pupil size from 6 mm to 3 mm.*

DISCUSSION

The aim of this study was to investigate the change in wave front refraction that is calculated on the basis of second order Zernike coefficients in dependence to the amount of fourth and sixth order aberrations. Even in normal eyes, a refraction measurement with 3 mm of pupil diameter can differ from that with 6 mm (within the same eye) over up to 1.4 D in sphere and up to 0.85 D in cylinder.

In theory, one has to study not only the amount of change in cylinder but also the change in the axis in dependence to the amount of higher order aberrations. In our study, we assumed that the change in the cylinder occurs always in the same axis as the original cylinder. This represents a worst case approximation for change in the amount of the cylinder. Nevertheless, one would observe a rather large change in the axis when the original cylinder is small compared to the amount of the fourth and sixth order astigmatism. However, the optical relevance of the induced cylinder might be small. In contrast, the change in axis would be small in case of a large cylinder and smaller higher order aberrations.

The focal length within an aberration-free optical system is given by the radius of curvatures of the given wave front. Introducing a defocus component simply shifts the location of the image plane along the optical axis of the eye. The amount of defocus is independent from the pupil size. Even in a normal eye one can observe a certain amount of higher order aberrations. This may lead to the situation that the calculation of the introduced defocus is dependent on the optical definition. In clinical use a high contrast acuity chart is used to perform subjective refraction. The visual system is doing an image plane analysis to optimize visual performance. But until know one does not know which reference plane for refractive calculation will fit as the best for the visual system. The paraxial focal plane represents the location of the image when only the central rays (close to the optical axis) are considered. The paraxial focal plane can be derived from the Taylor defocus and astigmatism terms appearing in equation (1). Here, only the central rays are considered and the peripheral rays are ignored. The mean spherical defocus, as it is calculated by the second order Zernike modes, represents the defocus in the case that the higher order aberrations are balanced – the wave front variance is minimized. One can also define the location of the area of least confusion to be the defocus of the particular eye. The marginal focus is formed by the marginal rays that enter the eye at the edge of the pupil and might be also used as a reference for defocus.

Guirao et al (2) recently published an article comparing two classes of methods for estimating the refractive state, one based on the wave front aberration defined in the pupil plane (similar to our study) and another based on image quality metrics valid in the retinal plane. Comparing the two principal strategies, these authors came to the conclusion that the refraction calculated on the basis of pupil plane aberrations is inferior at representing the subjective refraction in comparison to the refraction calculated on the basis of the image plane metrics. They concluded that higher order aberrations influence the amount of sphere and cylinder required to obtain optimal visual acuity, which is in accordance with our results.

The finding that higher order aberrations may affect a refractive error measured with objective methods such as autorefraction or retinoscopy implicates the question whether refraction values obtained from these methods represent a good estimate for subjective refraction. Siganos et al (18) correlated cycloplegic subjective refraction with cycloplegic autorefractometry in eyes that had laser *in situ* keratomileusis (LASIK). A statistically significant difference between subjective refraction and autorefraction was found in sphere and cylinder at all postoperative times. Only the axis was well correlated, thus we ignored the statistical analysis of the axis in our work. In the work of Siganos et al, the difference in refraction can be explained also in part by the difference in used aperture (subjective refraction 6 mm pupil size, autorefractor 2.5 to 3 mm sample size even in cycloplegic eyes) when performing the measurements.

Hament and coworkers (19) studied the repeatability of Zywave aberrometer measurements (Bausch & Lomb) and compared the measurements with subjective refraction and noncycloplegic and cycloplegic autorefractions in 20 eyes of 20 myopic patients. The wave front refractions were also calculated on the basis of low-order aberrations. In summary, they found that subjective refraction measurements are slightly more myopic than cycloplegic autorefraction measurements. In eyes with a dilated pupil, the Zywave measurements were significantly more myopic than subjective refractions and even more myopic than cycloplegic autorefractor readings. Zywave measurements and subjective refractions were in better agreement with a 3.5 mm pupil. These findings also support our results. In brief, the influence of higher order aberrations on the wave front refraction is small for a smaller pupil, thus, when comparing refraction data on the basis of a smaller pupil, one might estimate a better correlation than for larger pupils. Furthermore it should be kept in mind that comparing refraction data determined for a defined pupil size cannot be directly compared to refraction data determined for a significantly different pupil size (for example in dilated pupils).

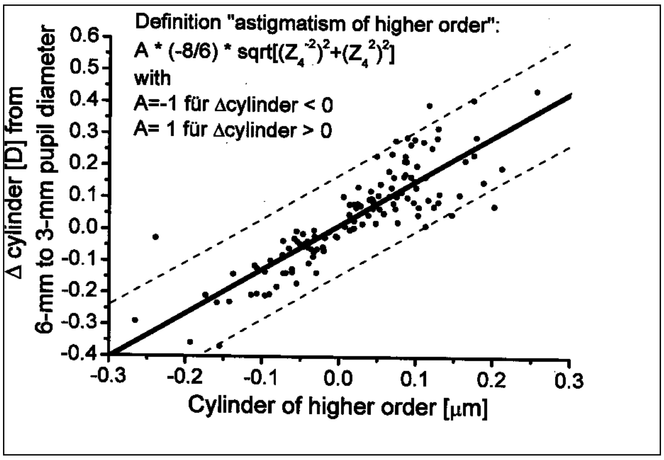


Fig. 5 - *Correlation between change in cylinder and higher order astigmatism. The correlation coefficient was R = 0.85 with a significance level of p<0.001.*

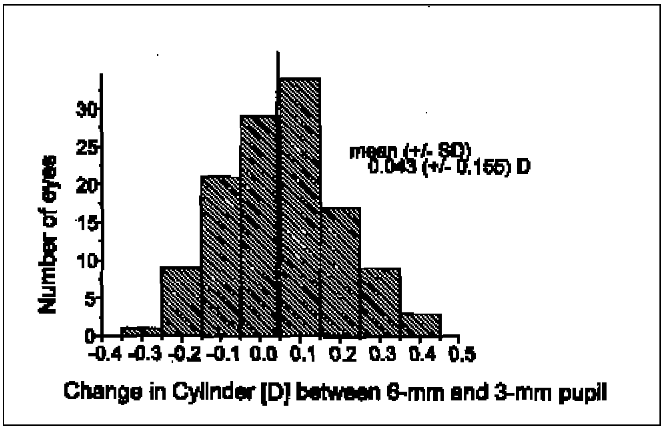


Fig. 6 - *Histogram of the differences in cylinder in all measured eyes when changing the pupil diameter from 3 to 6 mm. Approximately 95% of the investigated eyes have a change in cylinder up to 0.85 D when reducing the pupil size from 6 mm to 3 mm.*

Wang and associates (20) investigated the accuracy and reliability of a ray-tracing refractometer. Spherical and cylindrical refractive data calculated on the basis of Zernike coefficients correlated well with those derived from manifest refraction; however, there was a mean spherical error of approximately 1.10 D. Thus, they concluded that further work is required to refine the accuracy and range of the device that was used. However, our results and the results published by Guiaro and Williams (21) raise the question whether the spherical error is a result of mismatching the pupil size or the way the wave front refraction was calculated. In addition, one has to take into account possible accommodation during the measurement with a wave front device.

Thibos et al (22) found a significantly better correlation of the paraxial defocus with the subjective refraction compared to the wave front refraction based only on the second order Zernike terms. The differences between objective and subjective refractions for 6 mm pupils were –0.3±0.3 D (more myopic for mean objective refraction) and 0.0±0.3 D for minimum root-mean-square wave front error and paraxial refractions of the objective data, respectively.

The major deficiency of the present study is the inclusion of both partner eyes of an individual. It should be kept in mind that lower and higher order aberrations have mirror symmetry with a similar amount in both partner eyes (23, 24). However, the aim of our study was to investigate the clinical relevance of using second order Zernike polynomials for the calculation of wave front refraction data. Here, the mirror symmetry is of small relevance as the clinical data mainly serve to define the amount of possible errors associated with this method for refraction calculation.

In summary, our results demonstrate that the calculation of wave front refraction on the basis of low-order Zernike coefficients in the pupil plane is associated with a strong correlation to the higher order aberration preexistent in the subject eye. For clinical practice the use of wave front refraction based only on second Zernike coefficients is not recommended.

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